Low-Degree Graphs Beyond Planarity

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Problem

We study beyond planarity for graphs of low degree. In particular, we aim at establishing tight bounds for values of d such that every graph of degree at most d belongs to a certain beyond planarity class.

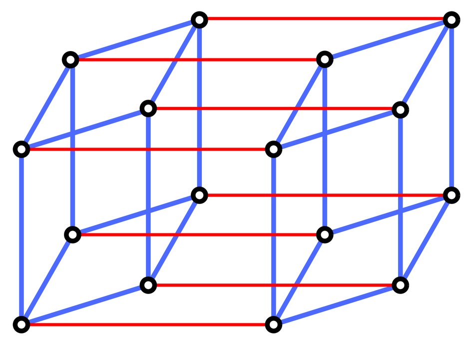
Left/right columns indicate largest/smallest d s.t. all (not all) degree-d graphs belong to a class.

Results

Theorem. For every $k \geq 1$, there exist bipartite Hamiltonian 3-regular graphs that are not k-planar.

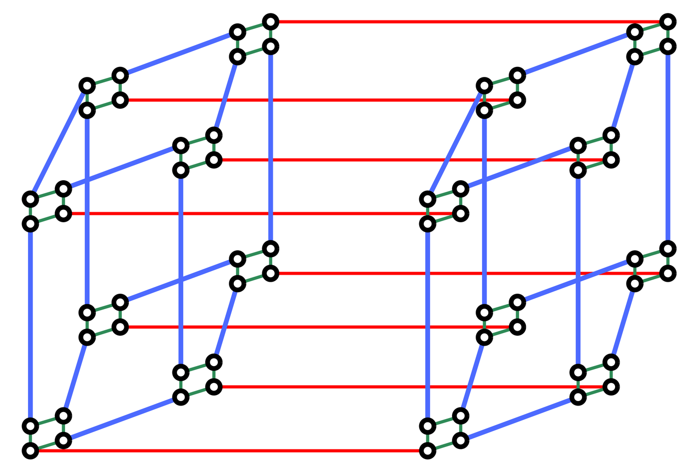
Proof: The 3-regular CCC_n graph [5,6] has $3n2^{n-1}$ edges and its crossing number is $> \frac{1}{20}4^n - (9n+1)2^{n-1}$. So, an edge has $> \lceil \frac{1}{15}\frac{2^n}{n} - 6 - \frac{2}{2n} \rceil$ crossings.

GD reviewer: Alternative proof through random graph theory.



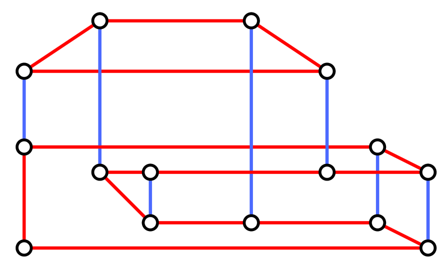
Hypercube graph Q_4

Cube-Connected Cycles CCC_4

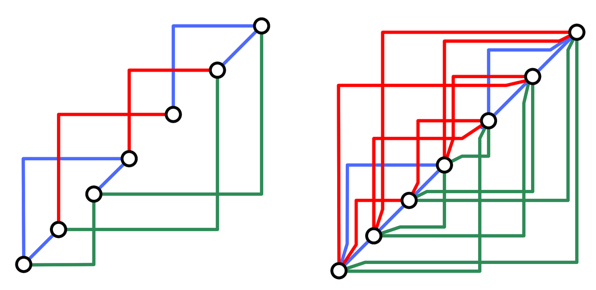


Planarity

Argyriou et al. [3]



Angelini et al. [2]



Alam et al. [1]

True for a set of cycles + a matching.

GD reviewer: Extended to all degree-3 graphs.

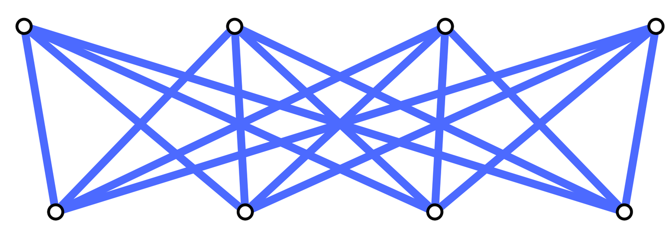
Feasible	Graph class	Infeasible
2	k-planar Ham. bipartite	3
2	fan-planar Ham. bipartite	3
2	RAC (0-bend)	4
3	RAC (0-bend) Ham.	4
3	RAC 1-bend	9
6	RAC 2-bend	148
3	fan-crossing-free	5
4	quasiplanar	10

Thickness 2

Theorem

The complete bipartite graph $K_{a,b}$, with $a \leq b$, is fan-crossing-free if and only if $a \in \{1, 2\}$, or $a \in \{3, 4\}$ and $b \leq 6$. In particular, $K_{5,5}$ is not fan-crossing-free.

Didimo et al. [4]



$K_{4,4}$

Density arguments

Applied to:

K_{10} (RAC 1-bend)
K_{149} (RAC 2-bend)
K_{11} (quasiplanar)

Open Problems

Narrow the gaps in the table:

- Are all degree-3 graphs RAC (0-bend)?
- Are all degree-4 graphs fan-crossing-free?
- Give negative examples for quasiplanarity and RAC 1-bend/2-bends not based on density arguments.

References

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[2] P. Angelini, L. Cittadini, W. Didimo, F. Frati, G. Di Battista, M. Kaufmann, and A. Symvonis. On the perspectives opened by right angle crossing drawings. J. Graph Algorithms Appl., 15(1):53-78, 2011.

[3] E. N. Argyriou, M. A. Bekos, M. Kaufmann, and A. Symvonis. Geometric RAC simultaneous drawings of graphs. J. Graph Algorithms Appl., 17(1):11-34, 2013.

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