Combining Problems on RAC Drawings and Simultaneous Graph Drawings*

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Abstract

We present an overview of the first combinatorial results for the so-called *geometric RAC simultaneous drawing problem*, i.e., a combination of problems on geometric RAC drawings [3] and geometric simultaneous graph drawings [1].

The GRACSim Problem

The *geometric RAC simultaneous drawing problem* (or *GRACSim*, for short) is stated as follows: Given two planar graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ with $E_1 \cap E_2 = \emptyset$, place their vertices on the plane so that, when the edges are drawn as straight-lines:

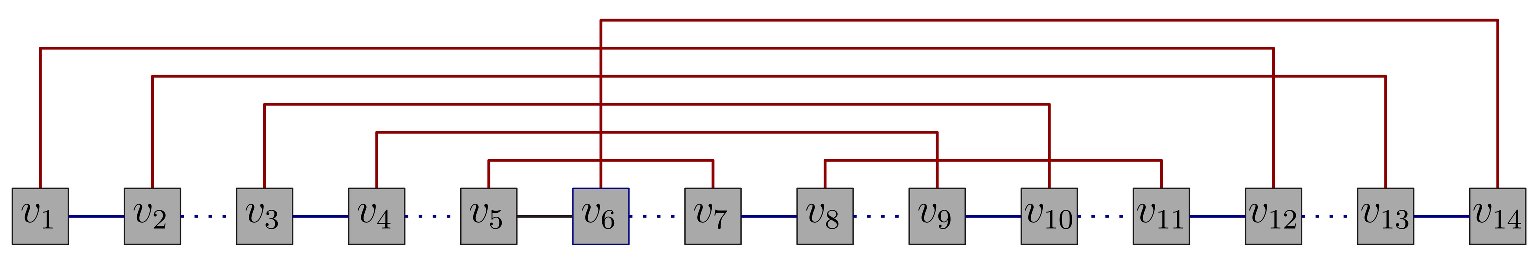
i. each graph is drawn planar
ii. there are no edge overlaps
iii. crossings between edges in E_1 and E_2 occur at right-angles.

Note: Graph $G_1 \cup G_2$ should be RAC. Thus: $|E_1 \cup E_2| < 4|V| - 10$.

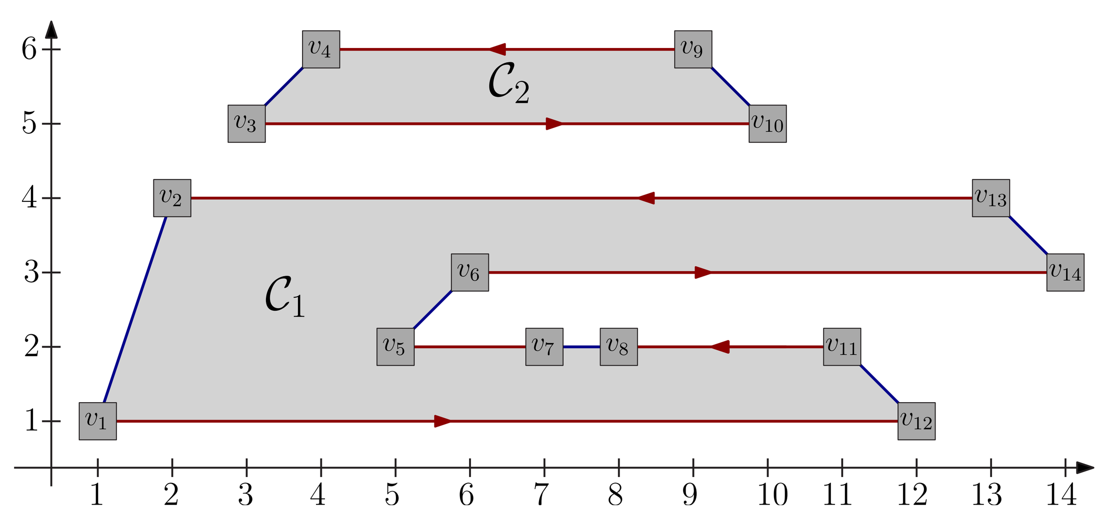
A Cycle and a Matching: A Positive Result

Theorem. *A cycle $\mathcal{C}$ and a matching $\mathcal{M}$ always admit a GRACSim drawing on an $(n+2) \times (n+2)$ integer grid. Moreover, the drawing can be computed in linear time.*

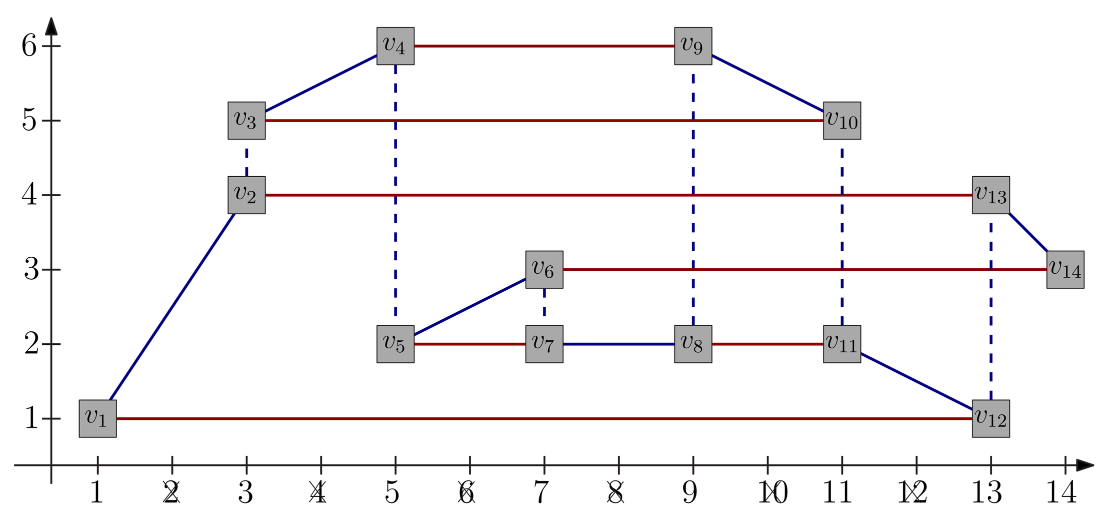
- If we remove an edge from $\mathcal{C}$, the remaining graph is a path $\mathcal{P}$.



- Identify in $\mathcal{P} \cup \mathcal{M}$ a cycle collection that contains half of $\mathcal{P}$'s edges and all of $\mathcal{M}$'s edges and draw it in a snake-like fashion.

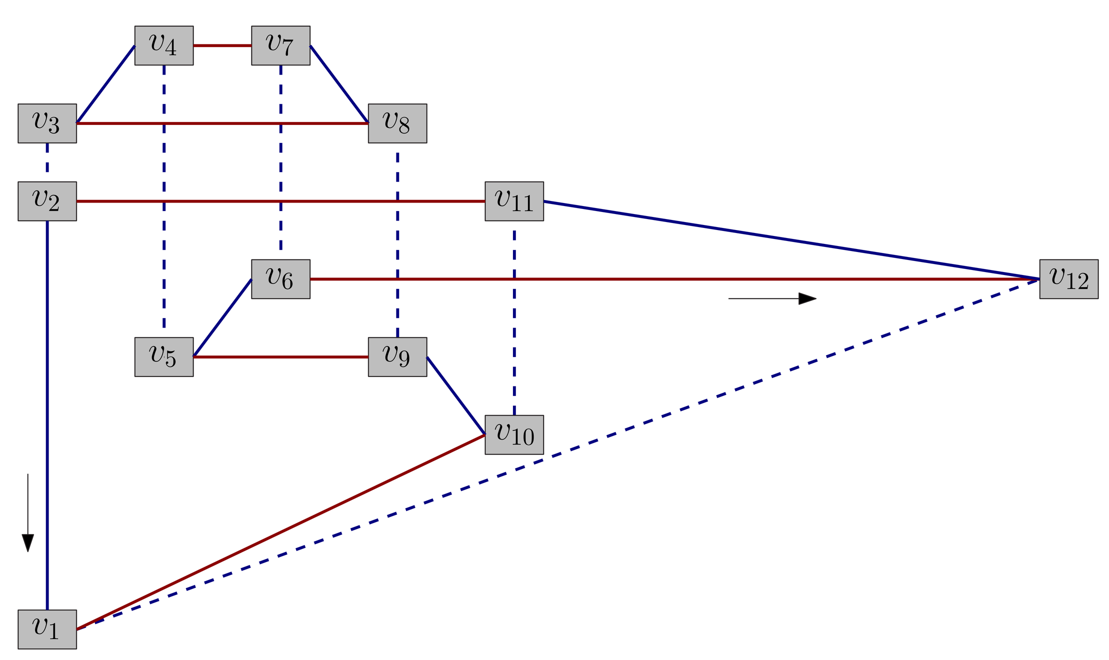


- Add the remaining edges of $\mathcal{P}$ and move each even-indexed vertex of $\mathcal{P}$ one unit to the right.



- Merge consecutive columns that do not interfere in y-direction into a common column.

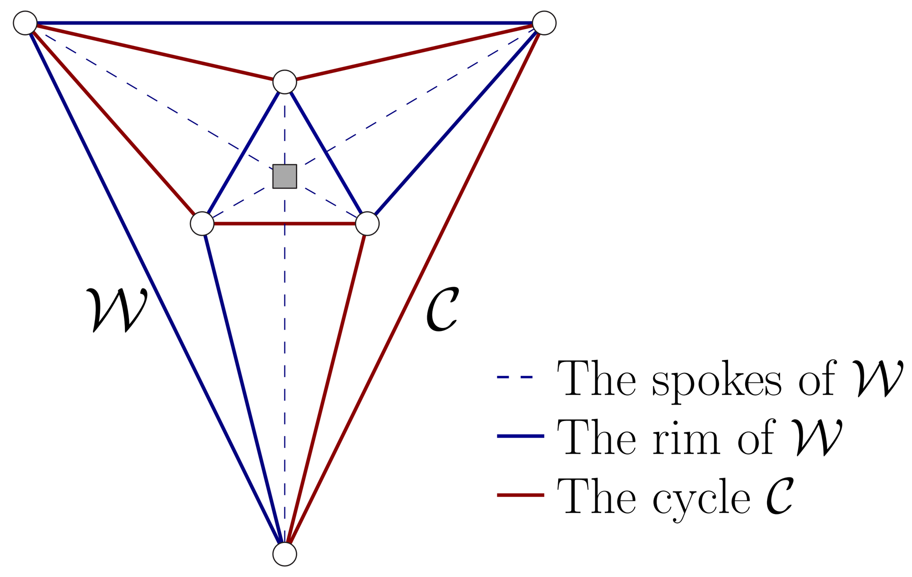
- Add the removed edge of $\mathcal{C}$.



A Wheel and a Cycle: A Negative Result

Theorem. *There exists a wheel $\mathcal{W}$ and a cycle $\mathcal{C}$ which do not admin a GRACSim drawing.*

- There is no RAC drawing of $\mathcal{W} \cup \mathcal{C}$ in which $\mathcal{W}$ is drawn planar.



Since Cabello et al. [2] have shown that a geometric simultaneous drawing of a wheel and a cycle always exists, the theorem mentioned above implies that if two graphs always admit a geometric simultaneous drawing, it is not necessary that they also admit a GRACSim drawing.

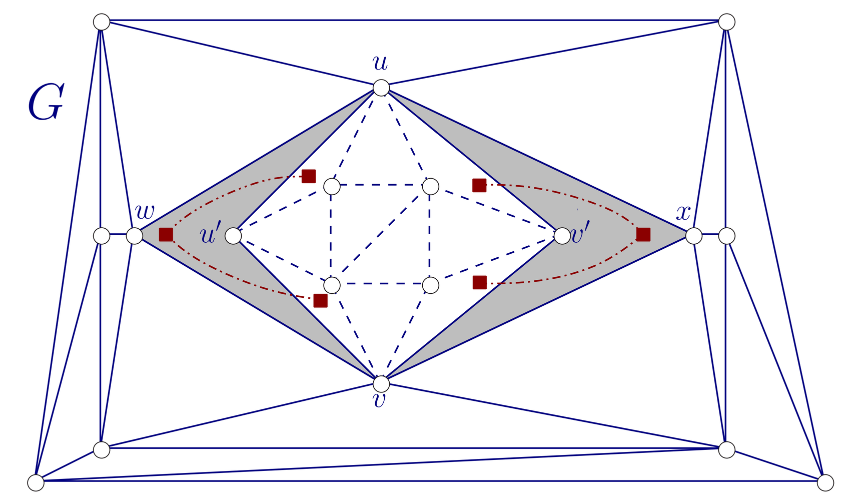
The GDual-GRACSim Problem: An Interesting Variation

According to the *GDual-GRACSim drawing problem*, we are given a planar embedded graph G and the main task is to determine a geometric drawing of G and its dual G^* (without the face-vertex corresponding to the external face) such that:

i. G and G^* are drawn planar
ii. each vertex of G^* is drawn inside its corresponding face of G
iii. the primal-dual edge crossings form right-angles.

Theorem. *Given a planar embedded graph G, a GDual-GRACSim drawing of G and its dual G^* does not always exist.*

- A graph that is a subdivision of a triconnected graph and it has two planar combinatorial embeddings.
- In order to have a RAC drawing of G and G^* both $uu'vw$ and $uv'vx$ must be convex, which is impossible.



Theorem. *Given an outerplane embedding of an outerplanar graph G, it is always feasible to determine a GDual-GRACSim drawing of G and its dual G^*.*

References

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